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C1 January 2010 (MA)

Q1) $y = x^4 + x^{1/3} + 3$

$\frac{dy}{dx} = 4x^3 + \frac{1}{3}x^{-2/3}$

$= 4x^3 + \frac{1}{3x^{2/3}}$

$= \boxed{4x^3 + \frac{1}{3(\sqrt[3]{x})^2}}$

Q2a) $(7+\sqrt{5})(3-\sqrt{5}) = 21 - 7\sqrt{5} + 3\sqrt{5} - 5$

$= \boxed{16 - 4\sqrt{5}}$

b) $\frac{7+\sqrt{5}}{3+\sqrt{5}} = \frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$

$= \frac{(7+\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$

$= \frac{16-4\sqrt{5}}{9-3\sqrt{5}+3\sqrt{5}-5}$

$= \frac{16-4\sqrt{5}}{4}$

$= \boxed{4-\sqrt{5}}$

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3a) $3x + 5y - 2 = 0$

$$5y = -3x + 2$$

$$y = -\frac{3}{5}x + \frac{2}{5}$$

$$\boxed{\text{Gradient} = -\frac{3}{5}}$$

b) A perpendicular line would have a gradient of $\frac{5}{3}$.

Equation of perpendicular line:

$$y - y_1 = m(x - x_1)$$

Since line passes through $(3,1)$, use 3 as x_1 and 1 as y_1:

$$y - 1 = \frac{5}{3}(x - 3)$$

$$3(y-1) = 5(x-3)$$

$$3y - 3 = 5x - 15$$

$$3y = 5x - 12$$

$$\boxed{y = \frac{5}{3}x - 4}$$

Q4) $\dfrac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}$, $x > 0$

$= 5x^{-\frac{1}{2}} + x^{3/2}$

$y = \int (5x^{-1/2} + x^{3/2})\,dx$

$= \dfrac{5x^{1/2}}{\frac{1}{2}} + \dfrac{x^{5/2}}{\frac{5}{2}} + C$

$= 10x^{1/2} + \dfrac{2x^{5/2}}{5} + C$

$= 10\sqrt{x} + \dfrac{2x^2\sqrt{x}}{5} + C$

Given $y = 35$ at $x = 4$,

$35 = 10(\sqrt{4}) + \dfrac{2(4)^2(\sqrt{4})}{5} + C$

$35 = 10(2) + \dfrac{2(16)(2)}{5} + C$

$35 = 20 + \dfrac{64}{5} + C$

$\therefore C = \dfrac{11}{5}$

$$\boxed{y = 10\sqrt{x} + \dfrac{2x^2\sqrt{x}}{5} + \dfrac{11}{5}}$$

5) $y - 3x + 2 = 0 \Rightarrow y = 3x - 2$ ①
$y^2 - x - 6x^2 = 0 \Rightarrow y^2 - x - 6x^2 = 0$ ②

Substitute ① into ②:

$$(3x-2)^2 - x - 6x^2 = 0$$

$$(3x-2)(3x-2) - x - 6x^2 = 0$$

$$9x^2 - 12x + 4 - x - 6x^2 = 0$$

$$3x^2 - 13x + 4 = 0$$

$$(3x-1)(x-4) = 0$$

Either $x = \frac{1}{3}$ or $x = 4$

Substitute into ① for y:

When $x = \frac{1}{3}$, $y = 3\left(\frac{1}{3}\right) - 2 = 1 - 2 \underline{= -1}$

When $x = 4$, $y = 3(4) - 2 = 12 - 2 \underline{= 10}$

Solution set: $\boxed{x = \frac{1}{3}, y = -1 \quad x = 4, y = 10}$

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Q6a) $y = \frac{(x+3)(x-8)}{x}, x > 0$

$y = \frac{x^2 - 8x + 3x - 24}{x}$

$y = \frac{x^2 - 5x - 24}{x}$

$y = \frac{x^2}{x} - \frac{5x}{x} - \frac{24}{x}$

$\underline{y = x - 5 - 24x^{-1}}$

$\frac{dy}{dx} = 1 + 24x^{-2}$

$= \boxed{1 + \frac{24}{x^2}}$

b) When $x=2$, $\frac{dy}{dx} = 1 + \frac{24}{2^2} = 1 + 6 \; \underline{= 7}$

Also, when $x=2$, $y = 2 - 5 - \frac{24}{2} \; \underline{= -15}$

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $(2, -15)$ and $m = 7$

$y - (-15) = 7(x - 2)$

$y + 15 = 7x - 14$

$\boxed{y = 7x - 29}$

Q7) First term, $a = £150$.
Common difference, $d = £10$

a) $U_n = a + (n-1)d$

$U_{10} = 150 + (10-1)10$

$U_{10} = 150 + 90$

$\boxed{U_{10} = £240}$

b) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(2(150) + (20-1)10)$

$S_{20} = 10(300 + 190)$

$S_{20} = 10 \times 490$

$\boxed{S_{20} = £4900}$

c) For Kevin, $S_{20} = £9800$. Given $d = £30$:

$9800 = 10(2a + (19)(30))$

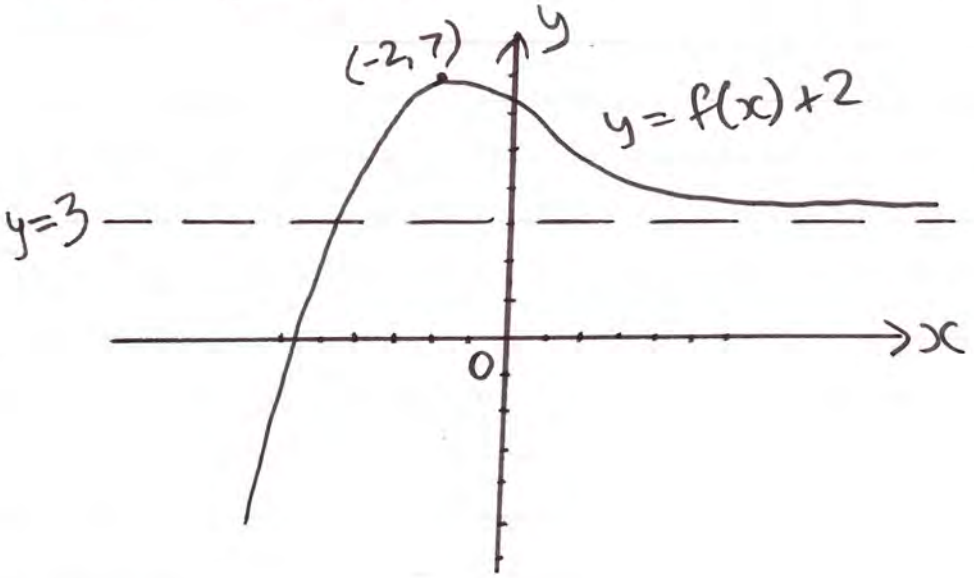
$9800 = 10(2a + 570)$

$9800 = 20a + 5700$

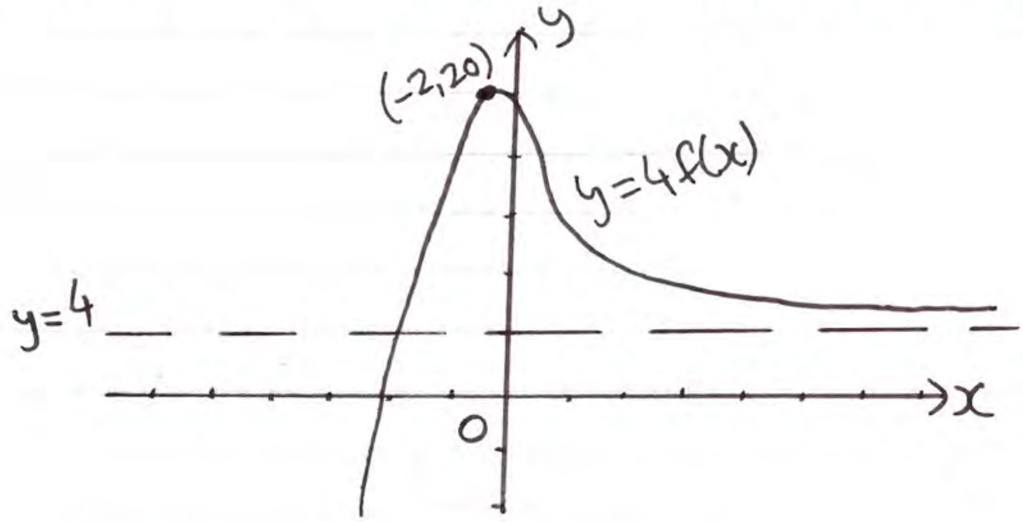
$20a = 4100$

$\therefore \boxed{a = £205}$

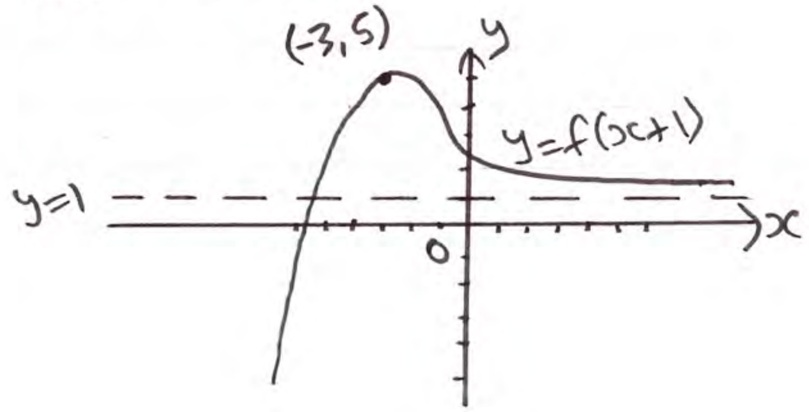
Q8a) $y = f(x) + 2$ - transformation of $+2$ along the y-axis:



b) $y = 4f(x)$ - multiply y-coordinates by 4:



c) $y = f(x+1)$ - transformation of -1 along the x-axis:



Q9a) $x^3 - 4x = x(x^2 - 4)$

$= \boxed{x(x+2)(x-2)}$

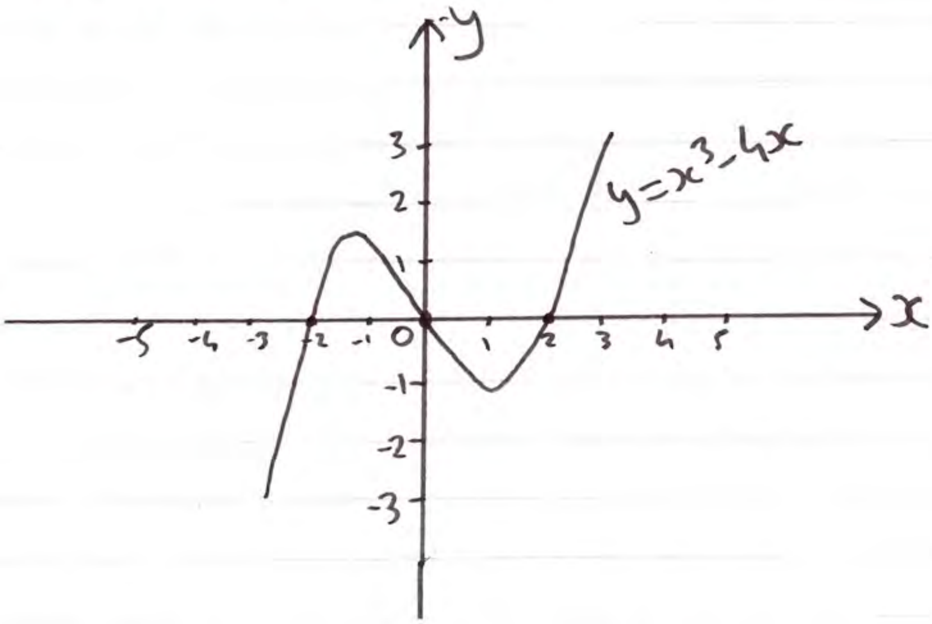
b) $y = x^3 - 4x$

When $x = 0, y = 0$

When $y = 0$, $x^3 - 4x = 0$

$x(x+2)(x-2) = 0$

Either $x = 0$ or $x = -2$ or $x = 2$



c) When $x = -1$, $y = (-1)^3 - 4(-1) = -1 + 4 = 3$

$\therefore$ A is at $(-1, 3)$

When $x = 3$, $y = 3^3 - 4(3) = 27 - 12 = 15$

$\therefore$ B is at $(3, 15)$

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Gradient $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{15-3}{3-(-1)}$

$= \frac{12}{4}$

$= 3$

Equation: $y - y_1 = m(x - x_1)$

Using $A(-1, 3)$ and $m = 3$ $\longrightarrow$ $y - 3 = 3(x - (-1))$

$y - 3 = 3(x+1)$

$y - 3 = 3x + 3$

$\boxed{y = 3x + 6}$

d) Length $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3-(-1))^2 + (15-3)^2}$

$= \sqrt{4^2 + 12^2}$

$= \sqrt{160} = \sqrt{16 \times 10} = \sqrt{16}\sqrt{10}$

$= \boxed{4\sqrt{10} \text{ units}}$

Q10a) $f(x) = x^2 + 4kx + (3+11k)$

$= (x+2k)^2 - (2k)^2 + (3+11k)$

$= (x+2k)^2 - 4k^2 + (3+11k)$

$= \boxed{(x+2k)^2 - 4k^2 + (3+11k)}$

b) If $f(x) = 0$ has no real roots, then the discriminant is less than 0.

$\therefore b^2 - 4ac < 0$

$(4k)^2 - (4)(1)(3+11k) < 0$

$16k^2 - 4(3+11k) < 0$

$16k^2 - 12 - 44k < 0$

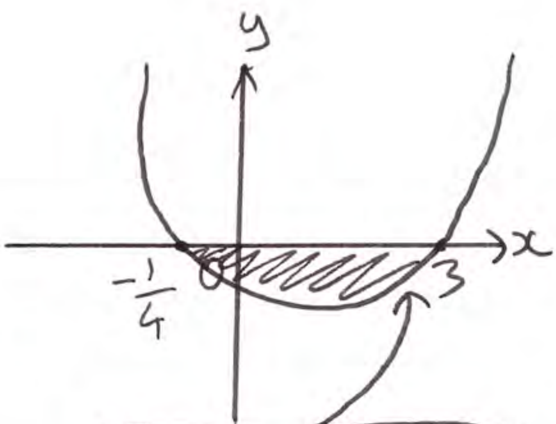
$4k^2 - 3 - 11k < 0$

$4k^2 - 11k - 3 < 0$

For $4k^2 - 11k - 3 = 0$,

$(4k+1)(k-3) = 0$

Either $k = -\frac{1}{4}$ or $k = 3$



Chosing values 'under' the x-axis since $b^2 - 4ac < 0$

Set of possible values of k:

$\boxed{-\frac{1}{4} < k < 3}$

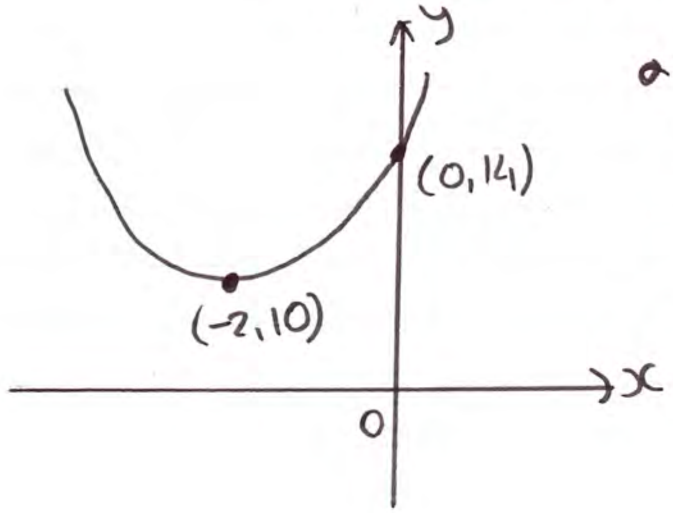
c) Given $k=1$

$f(x) = x^2 + 4x + 14$

When $x=0,\ f(x) = 14$

When $f(x)=0$, $x^2+4x+14=0$

No real roots, so curve does not cross x-axis



Turnig point at $-(2k), -4k^2 + (3+11k)$

when $k=1$,

Turning point is at $(-2, 10)$